Automated Planning

F3. Landmarks: Minimum Hitting Set Heuristic

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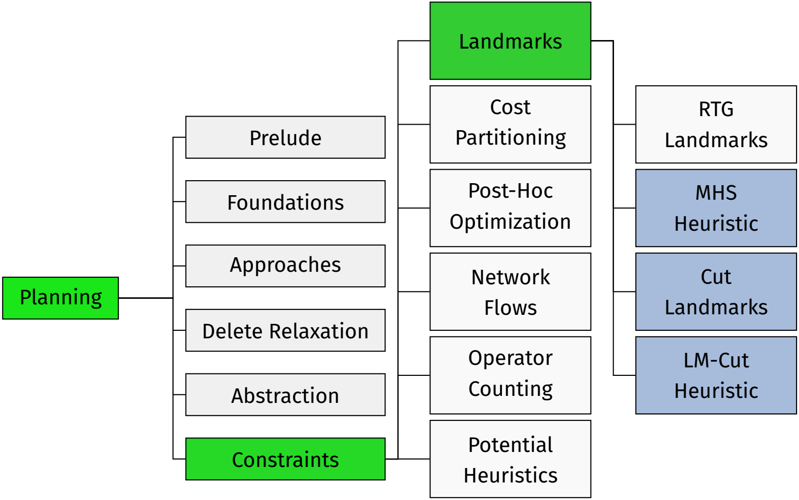
based on slides from the AI group at the University of Basel

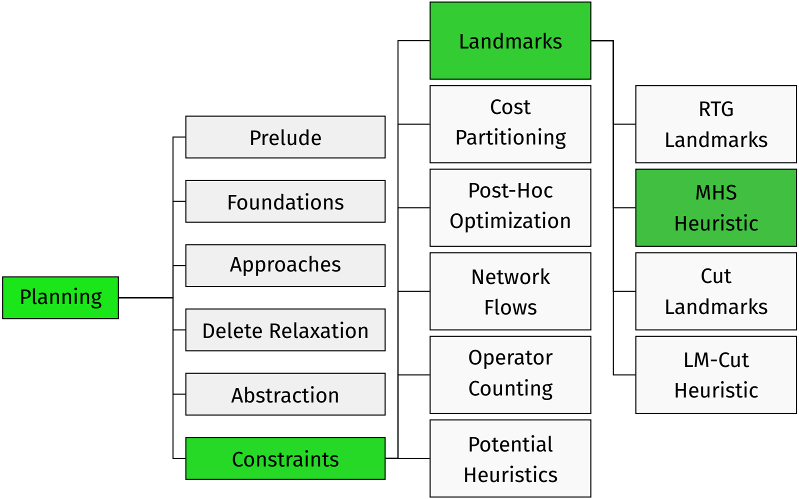
Content of this Course



The remaining landmark topics focus on disjunctive action landmarks.

Minimum Hitting Set Heuristic

Content of this Course



Exploiting Disjunctive Action Landmarks

- The cost $cost(L)$ of a disjunctive action landmark L is an admissible heuristic, but it is usually not very informative.
- Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ usually very weak heuristic
- better: Hitting sets

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \longrightarrow \mathbb{R}_0^+$ be a cost function for X.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of H is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3,\ c(o_2) = 4,\ c(o_3) = 5,\ c(o_4) = 0$

Specify a minimum hitting set.

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3,\ c(o_2) = 4,\ c(o_3) = 5,\ c(o_4) = 0$

Specify a minimum hitting set.

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

Definition (Hitting Set Heuristic)

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\mathrm{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s. Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for s.

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information...
- ...but is NP-hard to compute.
- $\rightsquigarrow$ Use approximations that can be efficiently computed.
  $\Rightarrow$ LP-relaxation, cost partitioning (both discussed later)

Summary

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- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks.
- The computation of a minimal hitting set is NP-hard.