#### Automated Planning

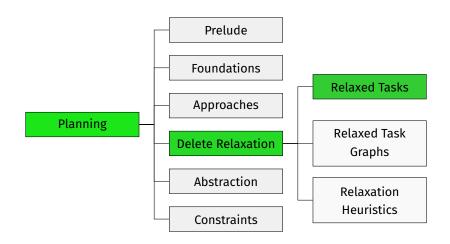
#### D2. Delete Relaxation: Finding Relaxed Plans

Jendrik Seipp

Linköping University

based on slides from the AI group at the University of Basel

#### Content of this Course



# **Greedy Algorithm**

#### **On-Set and Dominating States**

#### Definition (On-Set)

The on-set of an interpretation *s* is the set of propositional variables that are true in *s*.

 → for states of propositional planning tasks: states can be viewed as sets of (true) state variables

#### Definition (Dominate)

An interpretation s' dominates an interpretation s if  $on(s) \subseteq on(s')$ .

ightarrow all state variables true in s are also true in s'

## Motivation

- A general way to come up with heuristics is to solve a simplified version of the real problem.
- delete relaxation: given a task in positive normal form, discard all delete effects
- relaxation lemma: solutions for a state s also work for any dominating state s' (which satisfies a superset of the variables in s)
- monotonicity lemma: s[[o]] dominates s

## Greedy Algorithm for Relaxed Planning Tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

```
Greedy Planning Algorithm for \langle V, I, O^+, \gamma \rangle
s := 1
\pi^+ := \langle \rangle
loop forever:
     if s \models \gamma:
           return \pi^+
     else if there is an operator o^+ \in O^+ applicable in s
              with s[o^+] \neq s:
           Append such an operator o^+ to \pi^+.
           s := s [0^+]
     else:
```

return unsolvable

## Correctness of the Greedy Algorithm

The algorithm is sound:

- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable
  - Upon termination, there clearly is no relaxed plan from s.
  - By iterated application of the monotonicity lemma, s dominates *I*.
  - By the relaxation lemma, there is no solution from *I*.

What about completeness (termination) and runtime?

- Each iteration of the loop adds at least one atom to on(s).
- This guarantees termination after at most |V| iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
  - A good implementation runs in  $O(\|\Pi\|)$ .

## Using the Greedy Algorithm as a Heuristic

We can apply the greedy algorithm within heuristic search for a general (non-relaxed) planning task:

- When evaluating a state s in progression search, solve relaxation of planning task with initial state s.
- When evaluating a subgoal φ in regression search, solve relaxation of planning task with goal φ.
- Set h(s) to the cost of the generated relaxed plan.
  - in general not well-defined:
     different choices of o<sup>+</sup> in the algorithm lead to different h(s)

Is this admissible/safe/goal-aware/consistent?

## Using the Greedy Algorithm as a Heuristic

We can apply the greedy algorithm within heuristic search for a general (non-relaxed) planning task:

- When evaluating a state s in progression search, solve relaxation of planning task with initial state s.
- When evaluating a subgoal φ in regression search, solve relaxation of planning task with goal φ.
- Set h(s) to the cost of the generated relaxed plan.
  - in general not well-defined:
     different choices of o<sup>+</sup> in the algorithm lead to different h(s)

#### Is this admissible/safe/goal-aware/consistent?

It is safe (from the previous slide) and easily seen to be goal-aware. It is not admissible (see next slide) and therefore also not consistent.

## Properties of the Greedy Algorithm as a Heuristic

Is this an admissible heuristic?

- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because the greedy algorithm can make poor choices of which operators to apply.

How hard is it to find optimal relaxed plans?

# **Optimal Relaxed Plans**

## **Optimal Relaxation Heuristic**

#### Definition ( $h^+$ heuristic)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task in positive normal form with states *S*.

The optimal delete relaxation heuristic  $h^+$  for  $\Pi$ is the function  $h : S \to \mathbb{R}^+_0 \cup \{\infty\}$ where h(s) is the cost of an optimal relaxed plan for s, i.e., of an optimal plan for  $\Pi^+_s = \langle V, s, O^+, \gamma \rangle$ .

(can analogously define a heuristic for regression)

 $h^+$  is admissible, safe, goal-aware, and consistent.

## Complexity of Optimal Relaxed Planning (1)

#### Theorem (Complexity of Optimal Relaxed Planning)

The BCPLANEX problem restricted to delete-relaxed planning tasks is NP-complete.

## Summary

#### Summary

- Because of their monotonicity property, delete-relaxed tasks can be solved in polynomial time by a greedy algorithm.
- However, the solution quality of this algorithm is poor.
- For an informative heuristic, we would ideally want to find optimal relaxed plans.
- The solution cost of an optimal relaxed plan is the estimate of the h<sup>+</sup> heuristic.
- However, the bounded-cost plan existence problem for relaxed planning tasks is NP-complete.