Automated Planning

B3. Formal Definition of Planning

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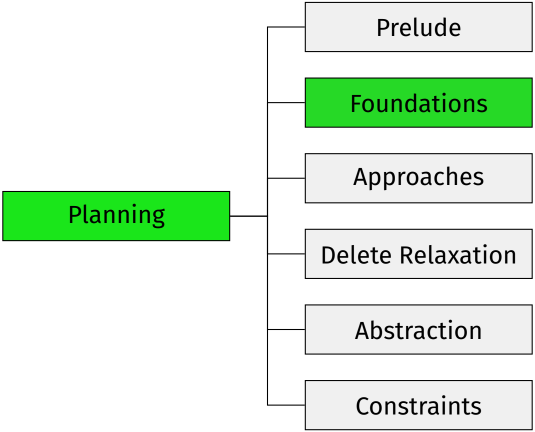
based on slides from the AI group at the University of Basel

Content of this Course



Semantics of Effects and Operators

Semantics of Effects: Effect Conditions

Definition (Effect Condition for an Effect)

Let ℓ be an atomic effect, and let e be an effect.

The effect condition $\mathit{effcond}(\ell, e)$ under which ℓ triggers given the effect e is a propositional formula defined as follows:

- $\mathit{effcond}(\ell, \top) = \bot$
- $\mathit{effcond}(\ell, e) = \top$ for the atomic effect $e = \ell$
- $\mathit{effcond}(\ell, e) = \bot$ for all atomic effects $e = \ell' \neq \ell$
- $\mathit{effcond}(\ell, (e \wedge e')) = (\mathit{effcond}(\ell, e) \vee \mathit{effcond}(\ell, e'))$
- $\mathit{effcond}(\ell, (\chi \rhd e)) = (\chi \wedge \mathit{effcond}(\ell, e))$

Intuition: $\mathit{effcond}(\ell, e)$ represents the condition that must be true in the current state for the effect e to lead to the atomic effect ℓ

Effect Condition: Example (1)

Example

Consider the move operator m_1 from the running example:
$\mathit{eff}(m_1) = ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))$.

Under which conditions does it set t_1 to false?

$$
\begin{aligned}
\mathit{effcond}(\neg t_1, \mathit{eff}(m_1)) &= \mathit{effcond}(\neg t_1, ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))) \\
&= \mathit{effcond}(\neg t_1, (t_1 \rhd \neg t_1)) \vee \\
&\quad\; \mathit{effcond}(\neg t_1, (\neg t_1 \rhd t_1)) \\
&= (t_1 \wedge \mathit{effcond}(\neg t_1, \neg t_1)) \vee \\
&\quad\; (\neg t_1 \wedge \mathit{effcond}(\neg t_1, t_1)) \\
&= (t_1 \wedge \top) \vee (\neg t_1 \wedge \bot) \\
&\equiv t_1 \vee \bot \\
&\equiv t_1
\end{aligned}
$$

Effect Condition: Example (2)

Example

Consider the move operator m_1 from the running example:
$\mathit{eff}(m_1) = ((t_1 \rhd \neg t_1) \land (\neg t_1 \rhd t_1))$.

Under which conditions does it set i to true?

$$
\begin{aligned}
\mathit{effcond}(i, \mathit{eff}(m_1)) &= \mathit{effcond}(i, ((t_1 \rhd \neg t_1) \land (\neg t_1 \rhd t_1))) \\
&= \mathit{effcond}(i, (t_1 \rhd \neg t_1)) \lor \\
&\ \mathit{effcond}(i, (\neg t_1 \rhd t_1)) \\
&= (t_1 \land \mathit{effcond}(i, \neg t_1)) \lor \\
&\ (\neg t_1 \land \mathit{effcond}(i, t_1)) \\
&= (t_1 \land \bot) \lor (\neg t_1 \land \bot) \\
&\equiv \bot \lor \bot \\
&\equiv \bot
\end{aligned}
$$

Semantics of Effects: Applying an Effect

first attempt:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$,
is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \\ s(v) & \text{otherwise} \end{cases}$$

What is the problem with this definition?

Semantics of Effects: Applying an Effect

correct definition:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$,
is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \land \neg \mathit{effcond}(v, e) \\ s(v) & \text{otherwise} \end{cases}$$

Add-after-Delete Semantics

Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of effect semantics is somewhat arbitrary, but has proven useful in applications.

Semantics of Operators

Definition (Applicable, Applying Operators, Resulting State)

Let V be a set of propositional state variables.
Let s be a state over V, and let o be an operator over V.

Operator o is applicable in s if $s \models pre(o)$.

If o is applicable in s, the resulting state of applying o in s, written $s[\![o]\!]$, is the state $s[\![eff(o)]\!]$.

Planning Tasks

Planning Tasks

Definition (Planning Task)

A (propositional) planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of propositional state variables,
- I is an interpretation of V called the initial state,
- O is a finite set of operators over V, and
- γ is a formula over V called the goal.

Running Example: Planning Task

Example

From the previous chapter, we see that the running example can be represented by the task $\Pi = \langle V, I, O, \gamma \rangle$ with

- $V = \{i, w, t_1, t_2\}$
- $I = \{i \mapsto \mathbf{F}, w \mapsto \mathbf{T}, t_1 \mapsto \mathbf{F}, t_2 \mapsto \mathbf{F}\}$
- $O = \{m_1, m_2, l_1, l_2, u\}$ where
  - $m_1 = \langle \top, ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1)), 5 \rangle$
  - $m_2 = \langle \top, ((t_2 \rhd \neg t_2) \wedge (\neg t_2 \rhd t_2)), 5 \rangle$
  - $l_1 = \langle \neg i \wedge (w \leftrightarrow t_1), (i \wedge w), 1 \rangle$
  - $l_2 = \langle \neg i \wedge (w \leftrightarrow t_2), (i \wedge \neg w), 1 \rangle$
  - $u = \langle i, \neg i \wedge (w \rhd ((t_1 \rhd w) \wedge (\neg t_1 \rhd \neg w))) \wedge (\neg w \rhd ((t_2 \rhd w) \wedge (\neg t_2 \rhd \neg w))), 1 \rangle$
- $\gamma = \neg i \wedge \neg w$

Exercise: Modeling a Propositional Planning Task

Example

Model the following task as a propositional planning task:

You are currently at home and have to write an essay. Since your computer is broken, you can only write the essay using a computer at the university library. These computers are always switched off when a user logs off.

Mapping Planning Tasks to Transition Systems

Definition (Transition System Induced by a Planning Task)

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces
the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- S is the set of all states over V,
- L is the set of operators O,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Planning Tasks: Terminology

- Terminology for transitions systems is also applied to the planning tasks Π that induce them.
- For example, when we speak of the states of Π, we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a solution of $\mathcal{T}(\Pi)$ is called a plan of Π.

Satisficing and Optimal Planning

By planning, we mean the following two algorithmic problems:

Definition (Satisficing Planning)

Given: a planning task Π

Output: a plan for Π, or **unsolvable** if no plan for Π exists

Definition (Optimal Planning)

Given: a planning task Π

Output: a plan for Π with minimal cost among all plans for Π, or **unsolvable** if no plan for Π exists

Positive Normal Form

Flat Effect

Definition (Flat Effect)

An effect is simple if it is either an atomic effect
or of the form $(\chi \rhd e)$, where e is an atomic effect.

An effect e is flat if it is a conjunction of 0 or more simple effects,
and none of these simple effects include the same atomic effect.

An operator o is flat if $\mathit{eff}(o)$ is flat.

Notes: analogously to CNF, we consider

- a single simple effect as a conjunction of 1 simple effect
- the empty effect as a conjunction of 0 simple effects

Flat Effect: Example

Example

Consider the effect

$$c \wedge (a \rhd (\neg b \wedge (c \rhd (b \wedge \neg d \wedge \neg a)))) \wedge (\neg b \rhd \neg a)$$

An equivalent flat (and conflict-free) effect is

$$\begin{aligned} & c \wedge \\ & ((a \wedge \neg c) \rhd \neg b) \wedge \\ & ((a \wedge c) \rhd b) \wedge \\ & ((a \wedge c) \rhd \neg d) \wedge \\ & ((\neg b \vee (a \wedge c)) \rhd \neg a) \end{aligned}$$

Note: if we want, we can write c as $(\top \rhd c)$ to make the structure even more uniform, with each simple effect having a condition.

Positive Formulas, Operators and Tasks

Definition (Positive Formula)

A logical formula φ is positive if no negation symbols appear in φ.

Note: This includes the negation symbols implied by $\rightarrow$ and $\leftrightarrow$.

Definition (Positive Operator)

An operator o is positive if $pre(o)$ and all effect conditions in $\mathit{eff}(o)$ are positive.

Definition (Positive Propositional Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is positive if all operators in O and the goal γ are positive.

Positive Normal Form

Definition (Positive Normal Form)

A propositional planning task is in positive normal form
if it is positive and all operator effects are flat.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$
\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}\} \\
I &= \{\mathit{home} \mapsto \mathbf{T}, \mathit{bike} \mapsto \mathbf{T}, \mathit{bike\text{-}locked} \mapsto \mathbf{T}, \\
&\qquad \mathit{uni} \mapsto \mathbf{F}, \mathit{lecture} \mapsto \mathbf{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \neg \mathit{bike\text{-}locked}, \neg \mathit{home} \land \mathit{uni}\rangle, \\
&\qquad \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \neg \mathit{bike\text{-}locked}\rangle, \\
&\qquad \langle \mathit{bike} \land \neg \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle, \\
&\qquad \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \neg \mathit{bike\text{-}locked}) \rhd \neg \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}
$$

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$
\begin{aligned}
V &= \{home, uni, lecture, bike, bike\text{-}locked\} \\
I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T}, \\
&\qquad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}\} \\
O &= \{\langle home \land bike \land \neg bike\text{-}locked, \neg home \land uni\rangle, \\
&\qquad \langle bike \land bike\text{-}locked, \neg bike\text{-}locked\rangle, \\
&\qquad \langle bike \land \neg bike\text{-}locked, bike\text{-}locked\rangle, \\
&\qquad \langle uni, lecture \land ((bike \land \neg bike\text{-}locked) \rhd \neg bike)\rangle\} \\
\gamma &= lecture \land bike
\end{aligned}
$$

Identify state variable v occurring negatively in conditions.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T}, \quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$$

$$O = \{\langle home \land bike \land \neg bike\text{-}locked, \neg home \land uni\rangle, \quad \langle bike \land bike\text{-}locked, \neg bike\text{-}locked\rangle, \quad \langle bike \land \neg bike\text{-}locked, bike\text{-}locked\rangle, \quad \langle uni, lecture \land ((bike \land \neg bike\text{-}locked) \rhd \neg bike)\rangle\}$$

$$\gamma = lecture \land bike$$

Introduce new variable $\hat{v}$ with complementary initial value.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$$
$$uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$$

$$O = \{\langle home \land bike \land \neg bike\text{-}locked, \neg home \land uni\rangle,$$
$$\langle bike \land bike\text{-}locked, \neg bike\text{-}locked\rangle,$$
$$\langle bike \land \neg bike\text{-}locked, bike\text{-}locked\rangle,$$
$$\langle uni, lecture \land ((bike \land \neg bike\text{-}locked) \rhd \neg bike)\rangle\}$$

$$\gamma = lecture \land bike$$

Identify effects on variable v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\}$$

$$I = \{\mathit{home} \mapsto \mathbf{T}, \mathit{bike} \mapsto \mathbf{T}, \mathit{bike\text{-}locked} \mapsto \mathbf{T}, \mathit{uni} \mapsto \mathbf{F}, \mathit{lecture} \mapsto \mathbf{F}, \mathit{bike\text{-}unlocked} \mapsto \mathbf{F}\}$$

$$\begin{aligned} O = \{&\langle \mathit{home} \land \mathit{bike} \land \neg\mathit{bike\text{-}locked}, \neg\mathit{home} \land \mathit{uni}\rangle, \\ &\langle \mathit{bike} \land \mathit{bike\text{-}locked}, \neg\mathit{bike\text{-}locked} \land \mathit{bike\text{-}unlocked}\rangle, \\ &\langle \mathit{bike} \land \neg\mathit{bike\text{-}locked}, \mathit{bike\text{-}locked} \land \neg\mathit{bike\text{-}unlocked}\rangle, \\ &\langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \neg\mathit{bike\text{-}locked}) \rhd \neg\mathit{bike})\rangle\} \end{aligned}$$

$$\gamma = \mathit{lecture} \land \mathit{bike}$$

Introduce complementary effects for $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T}, uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$$

$$O = \{\langle home \land bike \land \neg bike\text{-}locked, \neg home \land uni\rangle, \langle bike \land bike\text{-}locked, \neg bike\text{-}locked \land bike\text{-}unlocked\rangle, \langle bike \land \neg bike\text{-}locked, bike\text{-}locked \land \neg bike\text{-}unlocked\rangle, \langle uni, lecture \land ((bike \land \neg bike\text{-}locked) \rhd \neg bike)\rangle\}$$

$$\gamma = lecture \land bike$$

Identify negative conditions for v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, \textit{bike-locked}, \textit{bike-unlocked}\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, \textit{bike-locked} \mapsto \mathbf{T},$$
$$uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, \textit{bike-unlocked} \mapsto \mathbf{F}\}$$

$$O = \{\langle home \land bike \land \textit{bike-unlocked}, \neg home \land uni\rangle,$$
$$\langle bike \land \textit{bike-locked}, \neg \textit{bike-locked} \land \textit{bike-unlocked}\rangle,$$
$$\langle bike \land \textit{bike-unlocked}, \textit{bike-locked} \land \neg \textit{bike-unlocked}\rangle,$$
$$\langle uni, lecture \land ((bike \land \textit{bike-unlocked}) \rhd \neg bike)\rangle\}$$

$$\gamma = lecture \land bike$$

Replace by positive condition $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$
\begin{aligned}
V &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto \mathbf{T}, \mathit{bike} \mapsto \mathbf{T}, \mathit{bike\text{-}locked} \mapsto \mathbf{T}, \\
&\qquad \mathit{uni} \mapsto \mathbf{F}, \mathit{lecture} \mapsto \mathbf{F}, \mathit{bike\text{-}unlocked} \mapsto \mathbf{F}\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \mathit{bike\text{-}unlocked}, \neg \mathit{home} \land \mathit{uni}\rangle, \\
&\qquad \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \neg \mathit{bike\text{-}locked} \land \mathit{bike\text{-}unlocked}\rangle, \\
&\qquad \langle \mathit{bike} \land \mathit{bike\text{-}unlocked}, \mathit{bike\text{-}locked} \land \neg \mathit{bike\text{-}unlocked}\rangle, \\
&\qquad \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \mathit{bike\text{-}unlocked}) \rhd \neg \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}
$$

STRIPS

STRIPS Operators and Planning Tasks

Definition (STRIPS Operator)

An operator o of a propositional planning task is a STRIPS operator if

- $pre(o)$ is a conjunction of state variables, and
- $\mathit{eff}(o)$ is a conflict-free conjunction of atomic effects.

Definition (STRIPS Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is a STRIPS planning task if all operators $o \in O$ are STRIPS operators and γ is a conjunction of state variables.

STRIPS Operators: Remarks

- Every STRIPS operator is of the form

$$\langle v_1 \land \cdots \land v_n, \ \ell_1 \land \cdots \land \ell_m \rangle$$

  where v_i are state variables and ℓ_j are atomic effects.
- Often, STRIPS operators o are described via three sets of state variables:
  - the preconditions (state variables occurring in $pre(o)$)
  - the add effects (state variables occurring positively in $\mathit{eff}(o)$)
  - the delete effects (state variables occurring negatively in $\mathit{eff}(o)$)
- Definitions of STRIPS in the literature often do not require conflict-freeness. But it is easy to achieve and makes many things simpler.
- There exists a variant called STRIPS with negation where negative literals are also allowed in conditions.

Why STRIPS is Interesting

- STRIPS is particularly simple, yet expressive enough to capture general planning tasks.
- In particular, STRIPS planning is no easier than planning in general.
- Many algorithms in the planning literature are only presented for STRIPS planning tasks (generalization is often, but not always, obvious).

STRIPS

STanford Research Institute Problem Solver (Fikes & Nilsson, 1971)

Summary

Summary (1/2)

- Planning tasks compactly represent transition systems and are suitable as inputs for planning algorithms.
- A planning task consists of a set of state variables and an initial state, operators and goal over these state variables.
- In satisficing planning, we must find a solution for a planning task (or show that no solution exists).
- In optimal planning, we must additionally guarantee that generated solutions are of minimal cost.

Summary (2/2)

- A positive task with flat operators is in positive normal form.
- STRIPS is even more restrictive than positive normal form, forbidding complex preconditions and conditional effects.
- Both forms are expressive enough to capture general propositional planning tasks.
- Transformation to positive normal form is possible with polynomial size increase.
- Isomorphic transformations of propositional planning tasks to STRIPS can increase the number of operators exponentially; non-isomorphic polynomial transformations exist.