Finding Matrix Multiplication Algorithms with Classical Planning




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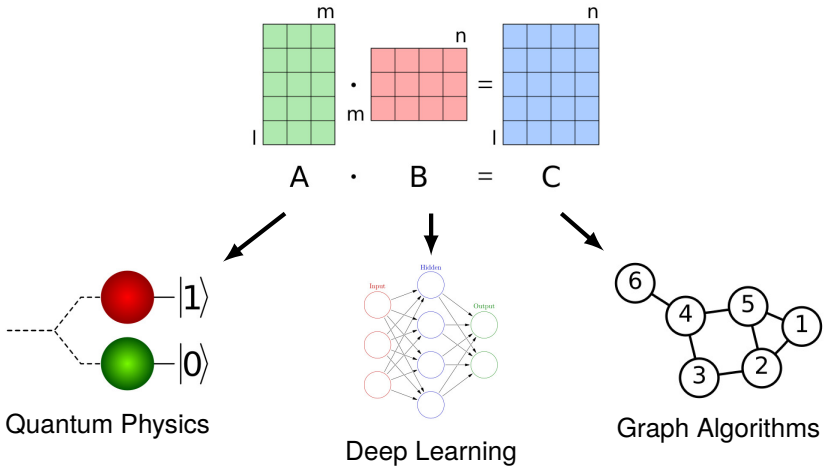
Matrix Multiplication – What is it good for?



How to Improve Matrix Multiplication?

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$

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$\Longrightarrow$ 8 multiplications

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Rich space of matrix multiplication algorithms can be formalized as decompositions of a specific 3D-tensor (Strassen 1969)!

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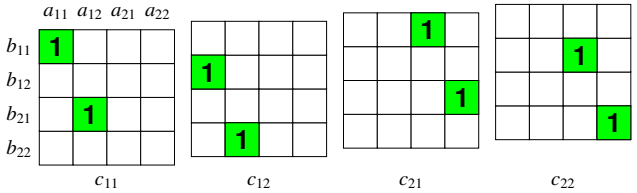
AlphaTensor: Reinforcement learning agent that discovered new and faster algorithms using MCTS (Fawzi et al. 2022) $\Longrightarrow$ Can we model it as a **classical planning task**?

Matrix Multiplication as Classical Planning

Strassen's Algorithm (in $\mathbb{Z}_2$)

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} = \begin{pmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{pmatrix}$$

$$\Downarrow$$



State

$$\mathbf{U} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

Actions

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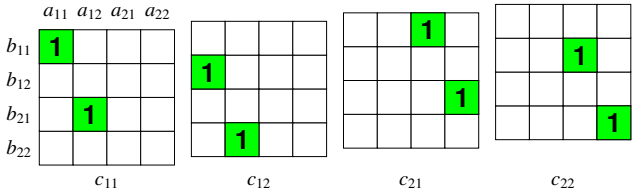

State

$$\mathbf{U} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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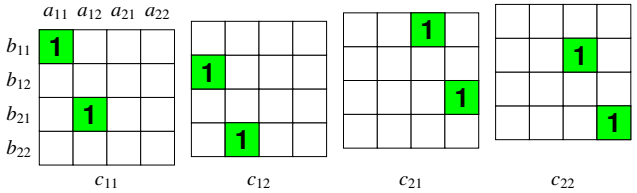

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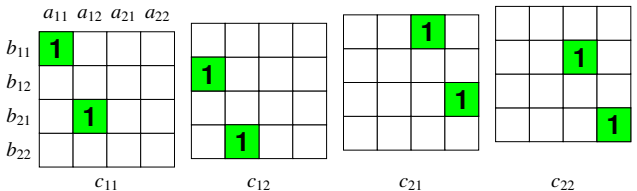

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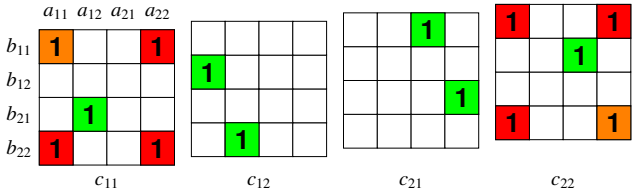

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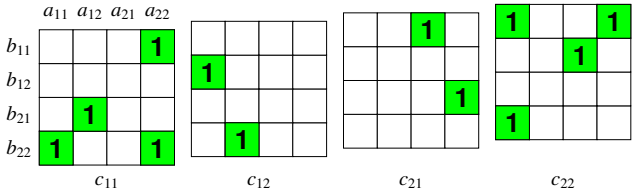

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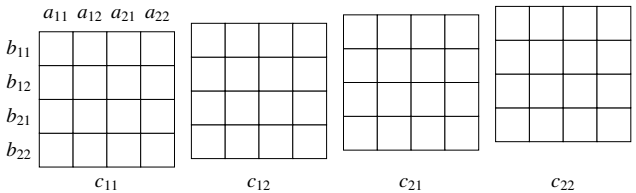

State

$$\mathbf{U} = \begin{bmatrix} 1\,0\,1\,0\,1\,1\,0 \\ 0\,0\,0\,0\,1\,0\,1 \\ 0\,1\,0\,0\,0\,1\,0 \\ 1\,1\,0\,1\,0\,0\,1 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1\,1\,0\,1\,0\,1\,0 \\ 0\,0\,1\,0\,0\,1\,0 \\ 0\,0\,0\,1\,0\,0\,1 \\ 1\,0\,1\,0\,1\,0\,1 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 1\,0\,0\,1\,1\,0\,1 \\ 0\,0\,1\,0\,1\,0\,0 \\ 0\,1\,0\,1\,0\,0\,0 \\ 1\,1\,1\,0\,0\,1\,0 \end{bmatrix}$$

Actions

Experiments

Setup

- 18 instances of MM problems
  - MM sizes: 1×1 to 3×3
  - Up to 134 million operators
- Optimal and satisficing planning
  - 4 planning systems
  - Different search techniques
- 10 hours and 80 GB on a single CPU core
- ${}^*X$: Concrete algorithm with X multiplications

Matrix Sizes	Rank			Satisficing				Optimal			
	L. Bound	U. Bound	Textbook	D–BFWS	LAMA	HC + h^{GC}	Lifted SAT	A* + h^{blind}	A* + h^{PDB}	Lifted SAT	Symbolic
111	1	1	1	*1	*1	*1	*101	*1	*1	*1	*1
⋮				⋮			⋮				⋮
132	6	6	6	*6	*6	*6	*8	4	*6	1	*6
213	6	6	6	*6	*6	*6	*6	4	*6	1	*6
222	7	7	8	*8	*8	*8	–	3	5	1	7
133	7	9	9	*9	*9	*9	–	3	4	1	7
⋮				⋮			⋮				⋮
323	7	15	18	–	*18	*18	–	2	2	–	3
333	19	23	27	–	–	*27	–	2	–	–	–

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⋮				⋮			⋮				⋮
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Conclusions

- Matrix Multiplication can be modeled as classical planning
- Encoding preserves completeness and optimality
- Off-the-shelf planners can tackle non-trivial instances
  - Interesting & challenging domain
- Confirmation of known bounds
- **Future Work:** Domain-specific pruning functions / heuristics

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Matrix Multiplication as Tensor Decomposition

Strassen's Algorithm (in $\mathbb{Z}_2$)

$$\mathbf{U} = \begin{bmatrix} 1&0&1&0&1&1&0 \\ 0&0&0&0&1&0&1 \\ 0&1&0&0&0&1&0 \\ 1&1&0&1&0&0&1 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1&1&0&1&0&1&0 \\ 0&0&1&0&0&1&0 \\ 0&0&0&1&0&0&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 1&0&0&1&1&0&1 \\ 0&0&1&0&1&0&0 \\ 0&1&0&1&0&0&0 \\ 1&1&1&0&0&1&0 \end{bmatrix}$$

$$m_1 = (a_{11} + a_{22})$$
$$m_2 = (a_{21} + a_{22})\, b_{11}$$
$$m_3 = a_{11}(b_{12} + b_{22})$$
$$m_4 = a_{22}(b_{21} + b_{11})$$
$$m_5 = (a_{11} + a_{12})b_{22}$$
$$m_6 = (a_{21} + a_{11})(b_{11} + b_{12})$$
$$m_7 = (a_{12} + a_{22})(b_{21} + b_{22})$$

$$c_{11} = m_1 + m_4 + m_5 + m_7$$
$$c_{12} = m_3 + m_5$$
$$c_{21} = m_2 + m_4$$
$$c_{22} = m_1 + m_2 + m_3 + m_6$$

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$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned}
m_1 &= (a_{11} + a_{22})\,(b_{11} + b_{22}) \\
m_2 &= (a_{21} + a_{22})\,b_{11} \\
m_3 &= a_{11}(b_{12} + b_{22}) \\
m_4 &= a_{22}(b_{21} + b_{11}) \\
m_5 &= (a_{11} + a_{12})b_{22} \\
m_6 &= (a_{21} + a_{11})(b_{11} + b_{12}) \\
m_7 &= (a_{12} + a_{22})(b_{21} + b_{22}) \\
c_{11} &= m_1 + m_4 + m_5 + m_7 \\
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c_{21} &= m_2 + m_4 \\
c_{22} &= m_1 + m_2 + m_3 + m_6
\end{aligned}$$

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$$\mathbf{W} = \begin{bmatrix} 1&0&0&1&1&0&1 \\ 0&0&1&0&1&0&0 \\ 0&1&0&1&0&0&0 \\ 1&1&1&0&0&1&0 \end{bmatrix}$$

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$$c_{11} = m_1 + m_4 + m_5 + m_7$$
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$$c_{22} = m_1 + m_2 + m_3 + m_6$$

References I

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