Managing Infinite Abstractions in Numeric Pattern Database Heuristics

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(Simple) Numeric Planning

Finite Domain Representations with Numbers

A Numeric Planning task is the tuple $\langle V_p \cup V_n, A, s_0, G\rangle$

- V_p: classic variables with finite domains
- V_n: numeric variables with values in $\mathbb{Q}$
- A: finite set of actions of the form $\langle \mathrm{pre}, \mathrm{eff}, \mathrm{cost}\rangle$, where $\mathrm{pre} = \mathrm{pre}_p \cup \mathrm{pre}_n$ (propositional and numeric conditions), and $\mathrm{eff} = \mathrm{eff}_p \cup \mathrm{eff}_n$ (propositional and numeric effects)
- s_0: initial state
- $G = G_p \cup G_n$: goal conditions

Simple Numeric Planning (SNP)

A state s is a tuple $\langle s_p, s_n\rangle$ where s is a full assignment over the variables in $V_n \cup V_p$.

- Define $\bar{\Psi}_n := G_n \cup \{\psi \mid \exists a \in A, \psi \in \mathrm{pre}_n(a)\}$ to be the finite set of all numeric conditions. Each condition ψ in $\bar{\Psi}_n$ has the form

$$\psi : \sum_{x \in V_n} w_x^\psi x \geq w_0^\psi .$$

- The numeric task is called simple since it has only simple effects of the form $(x := x + c_x^a)$ where $c_x^a \in \mathbb{Q}$.

Motivation. Current abstraction-based admissible heuristics (e.g., numeric PDBs and iPDB variants) still lag behind the state-of-the-art LM-cut heuristic on several numeric benchmarks. We aim to improve abstraction-based admissible heuristics for numeric planning by adapting and extending pattern database techniques to narrow the gap. Further research is required to finally close the gap.

PDBs for Simple Numeric Planning

PDBs (Classical)

- Let $P \subseteq V$ be a pattern and let $\Pi_P : S \to S|_P$ denote projection, i.e., restriction of states to a subset of variables.
- The PDB heuristic is the perfect heuristic in the projection:

$$h_{\mathrm{PDB}}(s) = h_P^*(\Pi_P(s))$$

- Notes: admissible (optimal plans with A^*), precomputed by backward/regression search, and combined by **max** or by **sum** when using pattern collections.

PDBs (Numeric)

- Projecting tasks with numeric variables typically yields an *infinite* abstract transition system since numeric variables have infinite domains.
- Practical recipe: grow a finite fragment from the abstract start via uniform-cost search; let S_E be expanded nodes and S_F the fringe.
- Finite-fragment heuristic (for $s \in S_E \cup S_F$):

$$h(s) = \min_{s' \in S_F} \{\, \mathrm{cost}^*(s, s') + d(s') \,\}, \quad d(s') = \begin{cases} 0 & s' \in S_* \\ \min \text{ action cost} & \text{otherwise.} \end{cases}$$

- Admissibility: distances in the (finite) image are lower bounds of concrete distances (homomorphism argument; see Gnad et al., 2025).
- Many abstract states remain unexpanded: use *min-action-cost* fallback for $s \notin S_E \cup S_F$.

Our Contribution

- A*: use exploration heuristic h_{Ex} to prioritize expansions (better than UCS for partial fragments).
- Failed-lookup h_{Fl}: lightweight, robust fallback when a projection is missing.

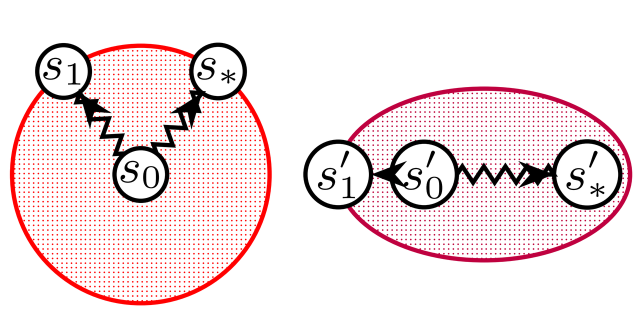

Figure 1: Illustration of heuristic values for s_0 and s_0' in uninformed (left) vs informed (right) partial expansion.

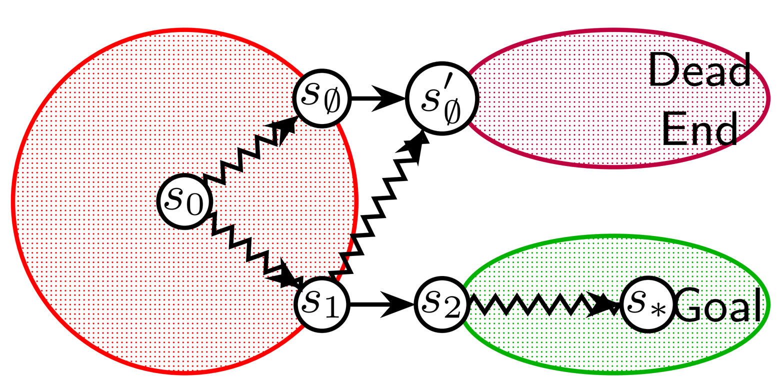

Figure 2: Detected dead ends can be missed by a partially-expanded PDB fragment.

Experimental Results

The XYZ Naming Scheme

- Stop A* exploration the abstract state space, guided by an *exploration* heuristic h_{Ex}, when a generation bound B is reached.
- For expanded states $s \in S_E$ we use a *fringe*-based refinement:

$$\tilde{h}(s) = \min_{s' \in S_F} \{\, \mathrm{cost}^*(s, s') + h_{\mathrm{Fr}}(s') \,\} \quad (s \in S_E),$$

replacing the prior min-action-cost fallback for unexpanded successors. *Naming:* This defines the *fringe* heuristic h_{Fr} used below in the XYZ naming.

- A failed-lookup heuristic h_{Fl} is used for states whose projection was not reached ($s|_P \notin S_E \cup S_F$).
- Notation: an instance is written as *XYZ*, where X, Y, Z select which heuristic variant is used for $h_{\mathrm{Ex}}, h_{\mathrm{Fr}}, h_{\mathrm{Fl}}$ (e.g., *BBB* baseline).

Coverage Results (Summary)

	Domain	#	B	L	BBB	LLB	BBL	BLL	LLL
IPC 2023	delivery	20	2	**3**	2	2	2	2	2
	drone	20	3	3	**4**	**4**	**4**	**4**	**4**
	expedition	20	5	**6**	**6**	**6**	**6**	**6**	**6**
	farmland-ipc23	15	4	**15**	8	**15**	**15**	**15**	**15**
	hydropower	20	9	**11**	9	9	8	10	10
	mprime	20	6	**15**	12	12	12	12	12
	rover-ipc23	20	**4**	**4**	**4**	**4**	**4**	**4**	**4**
	sailing-ipc23	20	0	**8**	0	0	0	0	0
	sugar	20	2	**12**	3	3	3	3	3
	zenotravel-ipc23	20	6	**8**	6	6	6	6	6
from literature	counters	20	3	**5**	**5**	**5**	**5**	**5**	**5**
	counters-sym	11	2	**11**	8	8	8	8	9
	depots	20	4	**7**	**7**	**7**	**7**	**7**	**7**
	depots-sym	20	4	**7**	6	6	6	6	6
	farmland	30	12	**30**	**30**	**30**	26	**30**	**30**
	fn-counters-small	8	6	**7**	**7**	**7**	**7**	**7**	**7**
	forestfire	20	10	**11**	10	10	10	10	10
	minecraft-pogo	20	14	5	**18**	17	**18**	17	17
	minecraft-sword	20	**20**	9	**20**	**20**	**20**	**20**	**20**
	petri-net	20	2	8	**9**	8	8	**9**	**9**
	plant-watering	63	**63**	**63**	**63**	**63**	**63**	**63**	**63**
	rover-unit	20	4	**7**	6	6	6	6	6
	sailing	40	10	**40**	15	18	15	17	17
	satellite	20	1	**2**	**2**	1	1	**2**	**2**
	zenotravel	23	6	**13**	10	9	10	10	10
	others	72	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	$\sum$	622	203	**311**	253	269	272	280	281

Table 1: B = Blind (UCS), L = LM-cut.

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