Isomorphism between STRIPS instances and sub-instances

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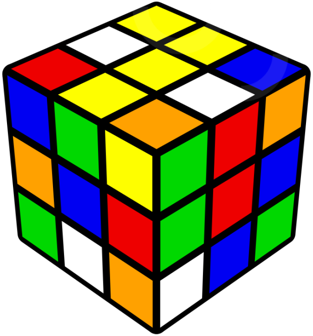
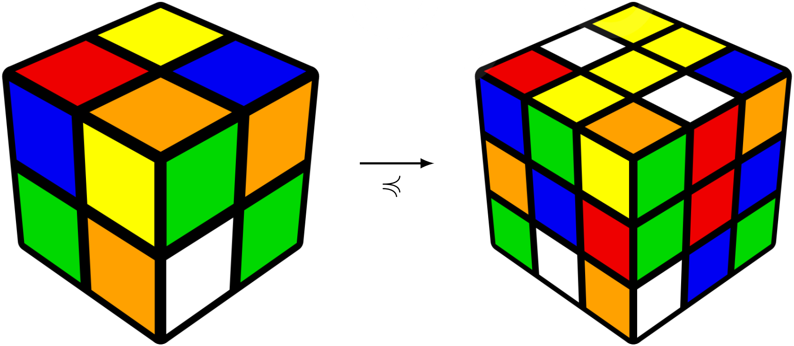
A simple example: Rubik's cube



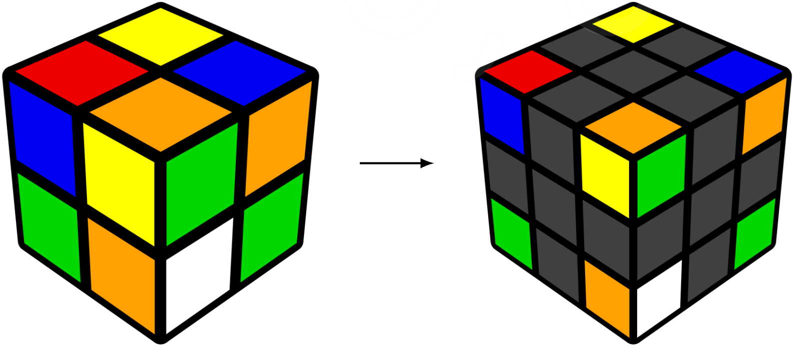
- Simple to describe
- 43e18 different configurations
- ...but "efficient" algorithms are known

A simple example: Rubik's cube



- What if we know how to solve a 3x3 cube...
- ...but not a 2x2?
- **Solution:** Reduce the problem to a 3x3 cube

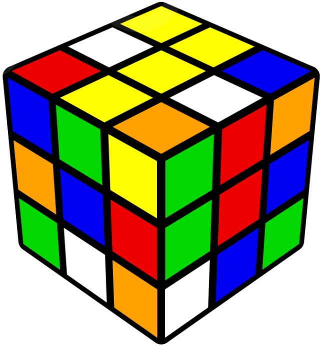
A simple example: Rubik's cube



- Map **elements** of the 2x2 cube to **elements** of the 3x3 cube
- Map **operations** on the 2x2 cube to **operations** on the 3x3

A simple example: Rubik's cubes

→ ?

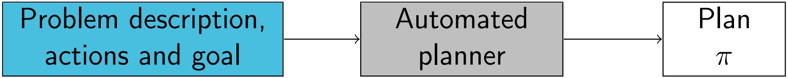


- Isomorphisms?
- Subinstance isomorphisms?
- No homomorphism exists?

Automated planning

Classical planning

- Model **of the problem** $\rightarrow$ **sequence of actions** π to solve it
- PSPACE-complete



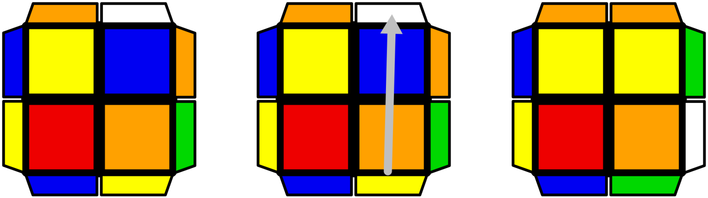
Topic of this talk: comparison between planning instances

1. Computational complexity
2. Algorithmic procedure

STRIPS instance: $P = \langle F, I, O, G \rangle$

- F: **fluents** (boolean variables that describe the current state)
- $I, G \subseteq F$: **initial** state and **goal**
- O: **operators** of the form:

$$o = \langle pre(o), eff^{+}(o), eff^{-}(o) \rangle \in (2^{F})^{3}$$



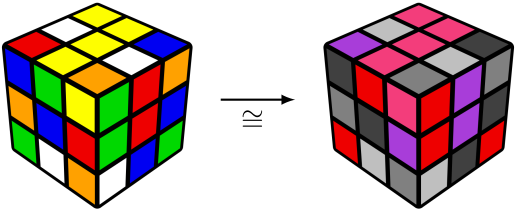
$$\left\{ \begin{array}{c} \text{col(up-top-right, blue),} \\ \text{col(up-low-left, red),} \\ \dots \end{array} \right\} \rightarrow \left\{ \begin{array}{c} \text{col(up-top-right, yellow),} \\ \text{col(up-low-left, red),} \\ \dots \end{array} \right\}$$

STRIPS Isomorphism

STRIPS Isomorphism - SI

Input: Two STRIPS instances P and $P' = \langle F', I', O', G' \rangle$
Output : $u : F \longrightarrow F'$ and $v : O \longrightarrow O'$ **one-to-one** s.t.

- For all $o \in O, \mathbf{v}(o) = \langle \mathbf{u}(pre(o)), \mathbf{u}(eff^{+}(o)), \mathbf{u}(eff^{-}(o)) \rangle$
- $I' = u(I)$
- $G' = u(G)$



STRIPS Isomorphism Problem SI - Complexity

SI is **GI-complete**

Complexity class GI

Problems polytime-reducible to the graph isomorphism problem

How hard is it?

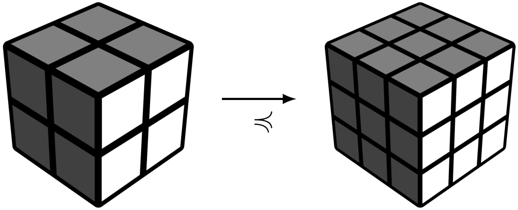
- **Theoretical:** $2^{O(\log n)^3}$ [Babai, 2016]
- **In practice:** Efficient solvers exist

Homogeneous STRIPS subinstance isomorphisms - SSI-H

Input: Two STRIPS instances P and $P' = \langle F', I', O', G' \rangle$

Output : Two functions $u : F \longrightarrow F'$ et $v : O \longrightarrow O'$ s.t.

- u **is injective**
- For all $o \in O, \mathbf{v}(o) = \langle \mathbf{u}(pre(o)), \mathbf{u}(eff^{+}(o)), \mathbf{u}(eff^{-}(o)) \rangle$



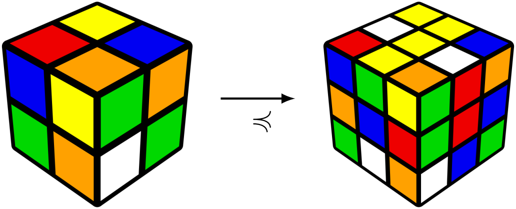
STRIPS subinstance isomorphisms

STRIPS subinstance isomorphism - SSI

Input: *(Same as above)*

Output: A homogeneous STRIPS subinstance isomorphism s.t.

- $I' = u(I)$
- $G' = u(G)$



STRIPS subinstance isomorphism SSI - Complexity

SSI is **NP-complete**

Proof idea

Reduction from Hamiltonian Cycle

Remark

Easier to find a subinstance isomorphism (NP-c) than to solve the planning problem (PSPACE-c)

Algorithm

1. Constraint propagation-based preprocessing
2. Compilation into SAT

Preprocessing

- **Aim:** Prune inconsistent mappings:
  - $f \in F \rightarrow f' \in F'$
  - $o \in O \rightarrow o' \in O'$

Idea

Maintain domains for the images of each fluents/actions, and prune them through *arc consistency*

Subroutine

Adapted from AC3

Fluents

- Initial domain chosen among these: (if applicable)
  - If $f \in \mathrm{I}$ then $\mathcal{D}(f) = \mathrm{I}'$
  - ...
  - Else $\mathcal{D}(f) = F' \setminus (I' \cup G')$

Actions

- Action profile: vector of $\mathbb{N}^k$
  - $\text{profile}(o) = \langle |\text{pre}(o)|, \ldots, |\text{eff}^-(o)|, |sd(o)| \rangle$
  - $sd(o)$: strict-delete fluents ($f \in \text{pre}(o) \wedge f \in \text{eff}^-(o)$)

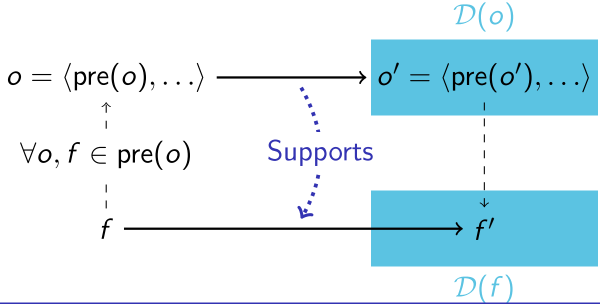
- $\mathcal{D}(o) = \{o' \mid \text{profile}(o') \geq \text{profile}(o)\}$

Prune **operators** not supported by some **fluent**

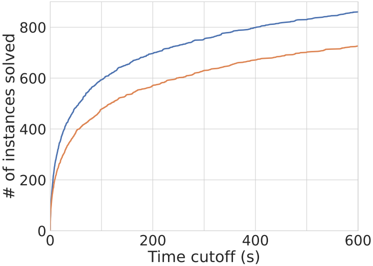
$$\mathcal{D}_v(o) \subseteq \{o' \mid \forall f \in pre(o), \exists f' \in pre(o') \text{ s.t. } f' \in \mathcal{D}_u(f) \wedge \ldots\}$$

Prune **fluents** not supported by some **operator**

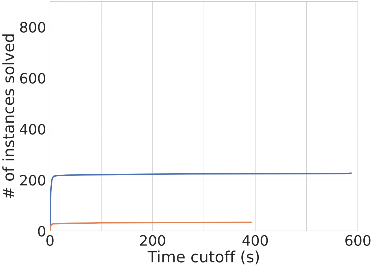
$$\mathcal{D}_u(f) \subseteq \left\{ f' \;\middle|\; \begin{array}{l} \forall o \in O \text{ where } f \in pre(o), \\ \exists o' \in \mathcal{D}_v(o) \text{ s.t. } f' \in pre(o') \\ \wedge \ldots \end{array} \right\}$$



Homogeneous **STRIPS subinstances solved before cutoff**



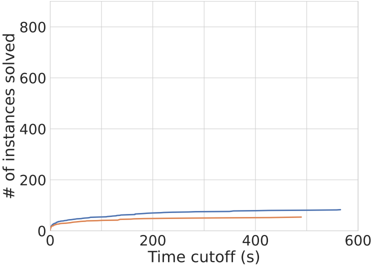
Positive SSI-H instances
(Subinstance iso. found)



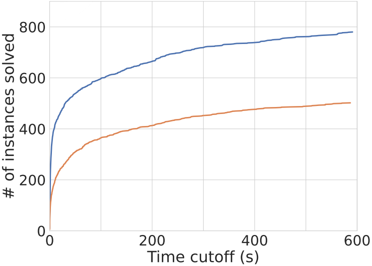
Negative SSI-H instances
(No subinstance iso. exists)

- Blue: Preprocessing on
- Orange: Preprocessing off

STRIPS subinstances solved before the time cutoff



Positive SSI instances
(Subinstance iso. found)



Negative SSI instances
(No subinstance iso. exists)

- Blue: Preprocessing on
- Orange: Preprocessing off

Observations - Preprocessing

Preprocessing step

- Lasts < 3 seconds in most cases
- Sometimes sufficient to detect unsolvable instances

Efficiency can vary with the domain

- Between 70% and $> 99\%$ of associations pruned (for 5 domains out of 7)
- $< 2\%$ of associations pruned (for 2 domains out of 7)

▶ Less efficient for domains with lots of symetries

Conclusion

Complexity

- Isomorphism: GI-complete
- Subinstance isomorphism: NP-complete

Algorithm

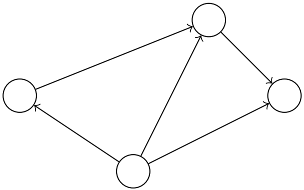
- Compilation to SAT
- Preprocessing: constraint propagation-based
- Feasible in practice

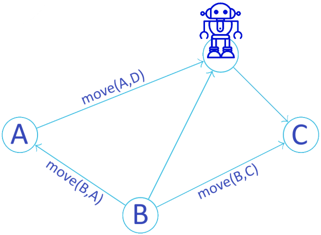
Future work:

- **Practical:** Exploit symmetries
- **Theoretical:**
  - Study properties carried over by homomorphisms
  - Find other forms of comparison relations

SI is GI-hard

- Reduction from the **graph** isomorphism problem





$\mathcal{G}(V, E) \quad \Rightarrow \quad P_{\mathcal{G}} = \langle V, I_{\mathcal{G}}, O_{\mathcal{G}}, G_{\mathcal{G}} \rangle$

SI is in GI

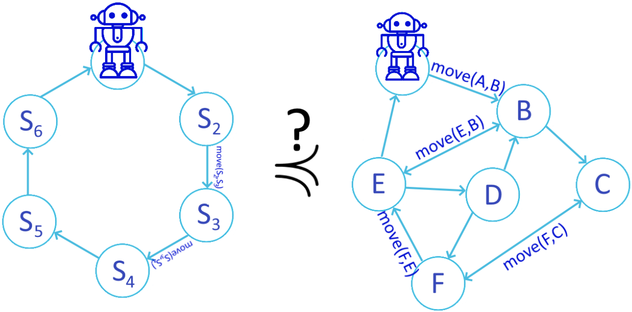
- Reduction to the **finite models** isomorphism problem $\mathcal{M} = \langle V, \mathcal{R}_1, \ldots, \mathcal{R}_n \rangle$

STRIPS subinstance isomorphism SSI - Complexity

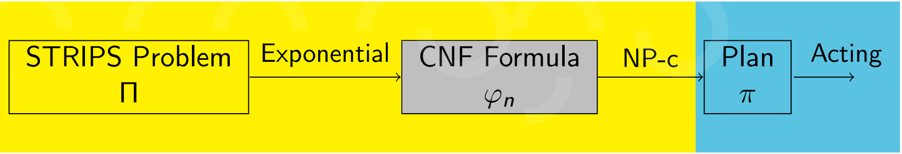
SSI is **NP-complete**

Reduction from Hamiltonian Cycle

- Encode input $\mathcal{G}(V, E)$ into STRIPS
- Encode a cycle graph of size $|V|$ into STRIPS
- A STRIPS subsintance isomorphism exists *iff* a cycle exists in $\mathcal{G}$



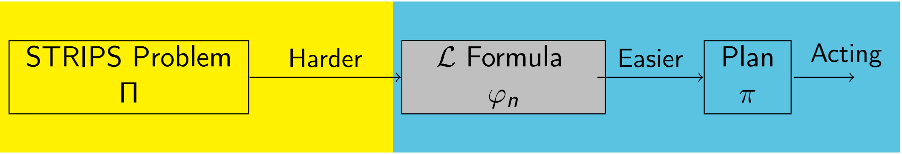
Planning as satisfiability [Kautz et al., 1992]



Offline Online

- φ_n: State-space encoded in CNF, up to n actions (**horizon** n)
- Bijection: models of $\varphi_n \Leftrightarrow$ plans of size n

Our Proposal



- $\mathcal{L}$: some propositional language to be determined

References

Babai, L. (2016).
Graph isomorphism in quasipolynomial time.

Kautz, H. A., Selman, B., et al. (1992).
Planning as satisfiability.
In *ECAI*, volume 92, pages 359–363. Citeseer.